



Universidade de Coimbra



Centro de Engenharia Mecânica  
da Universidade de Coimbra

# Remapping algorithms: application to trimming operations in sheet metal forming

**D.M. Neto<sup>1</sup> • C.M.A. Diogo<sup>1</sup> • T.F. Neves<sup>1</sup> • M.C. Oliveira<sup>1</sup> • J.L. Alves<sup>2</sup> •  
L.F. Menezes<sup>1</sup>**

<sup>1</sup>CEMUC, Department of Mechanical Engineering, University of Coimbra, Portugal

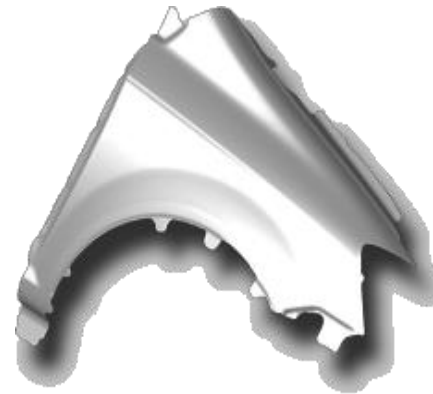
<sup>2</sup>CMEMS, Department of Mechanical Engineering, University of Minho, Portugal

# Sheet metal forming processes

- Multi-step sheet metal forming process
- Commonly adopted in the production of complex automotive structural components



Forming process

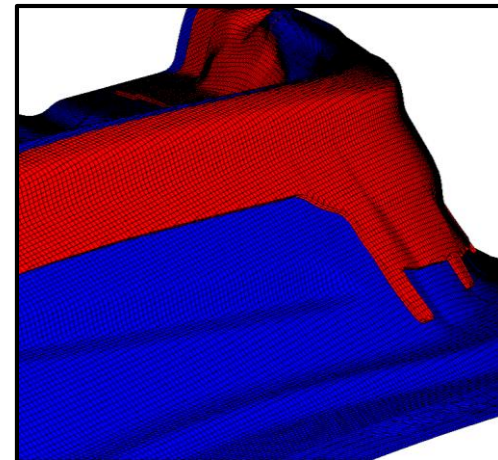
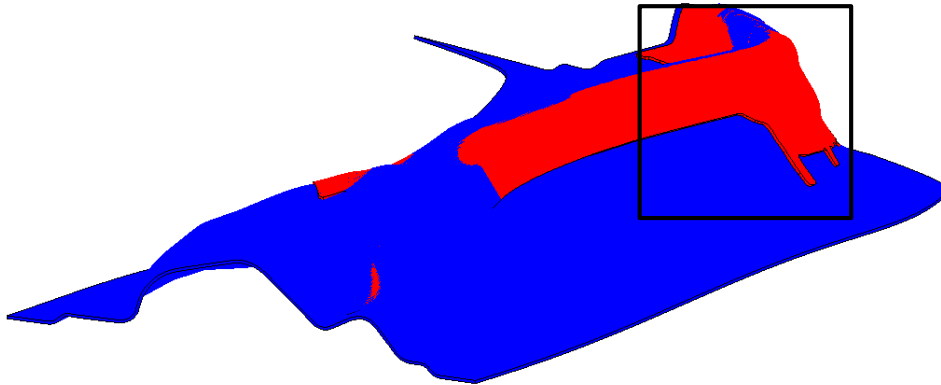


Trimming process

Front fender  
(Benchmark C - Numisheet 2002)

## Numerical simulation

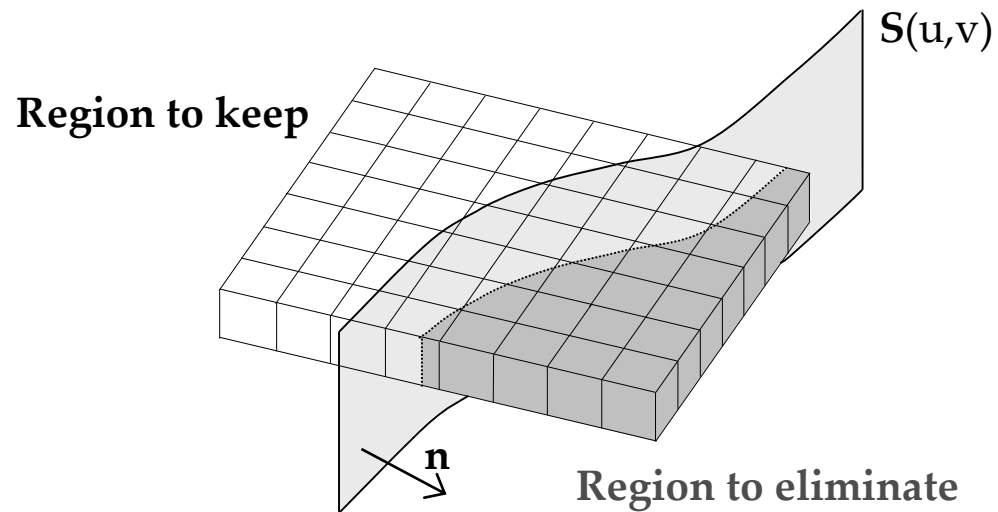
- Numerical simulation is an **indispensable tool** in the development of new automotive components manufactured by forming
- Finite element modelling of sheet metal forming processes is **very complex** due to the highly non-linear material behavior, large strains and frictional contact conditions
- Intermediate stages for trimming



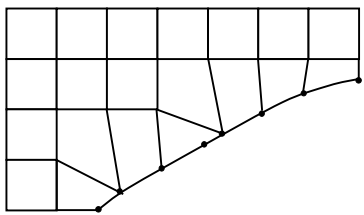
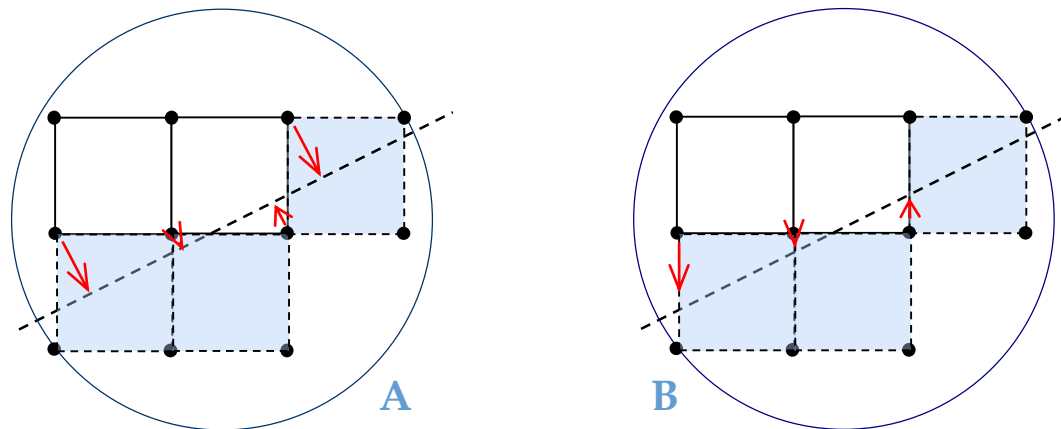
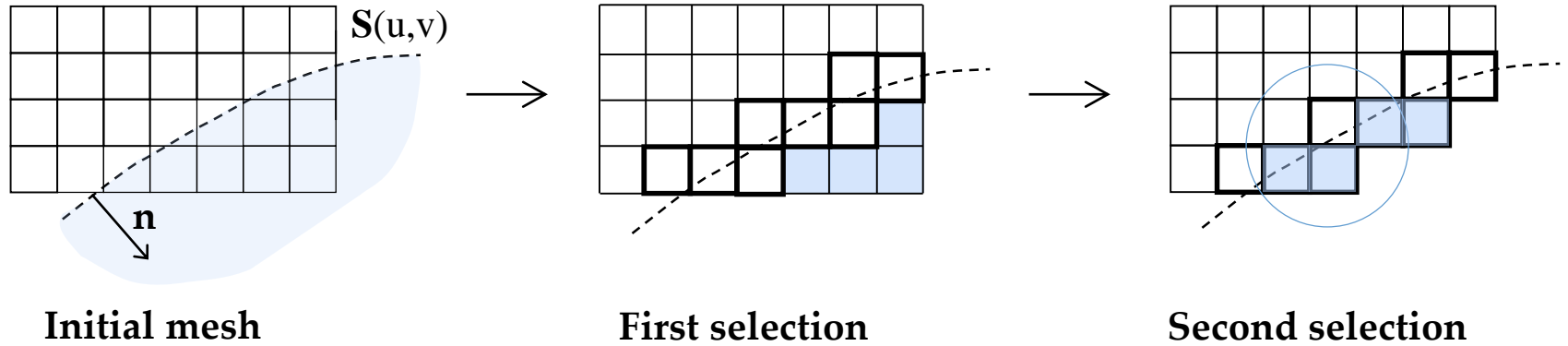
Aluminium Panel

(Benchmark 2 - Numisheet 2016)

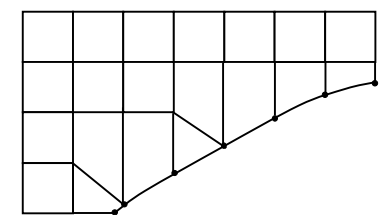
# Trimming



# Trimming

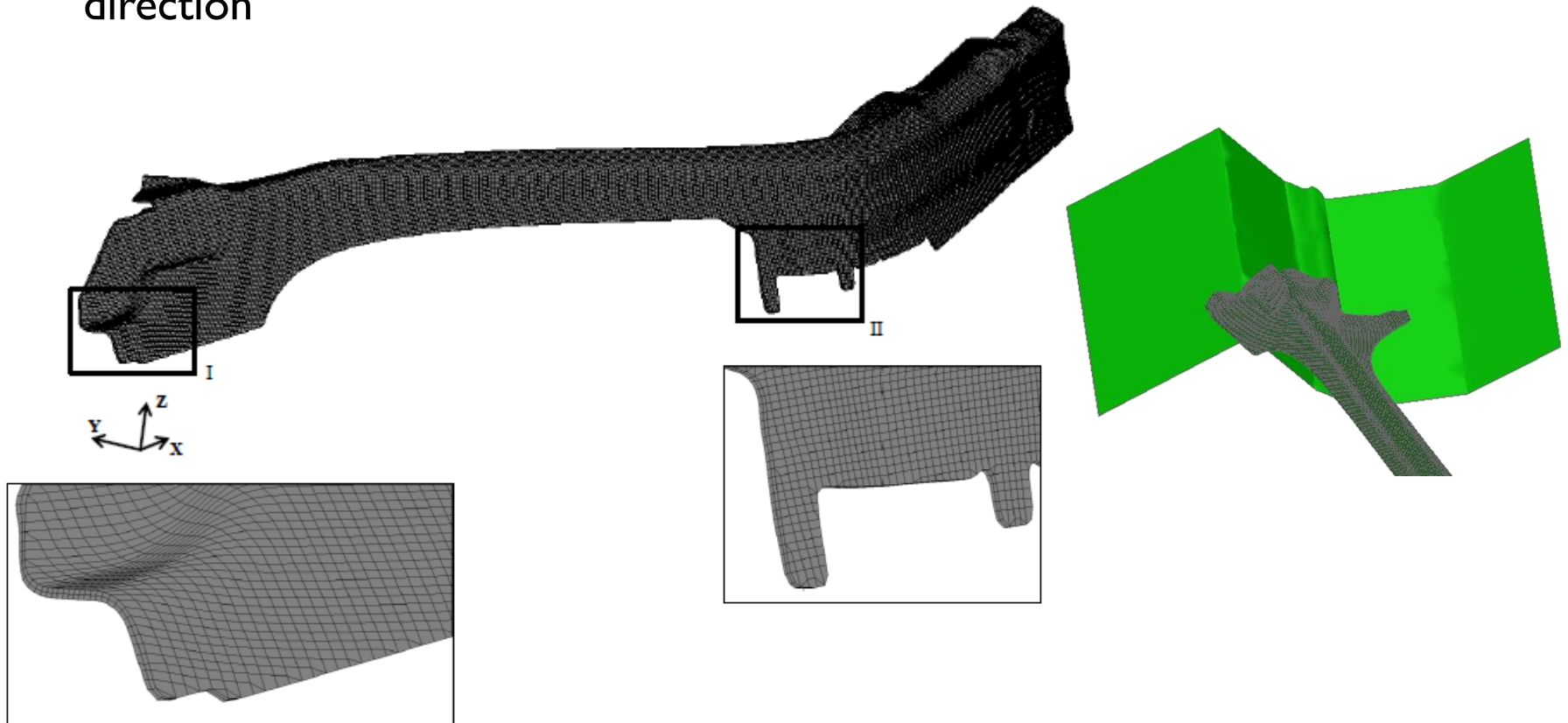


Correction



# Trimming

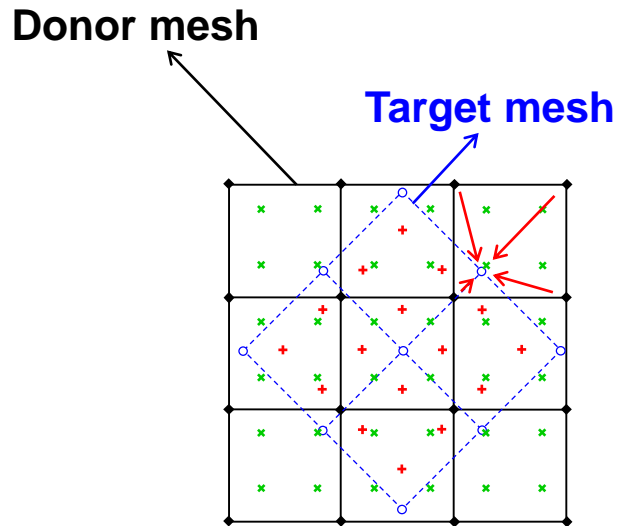
- Implies the definition of trimming surfaces properly aligned with the thickness direction



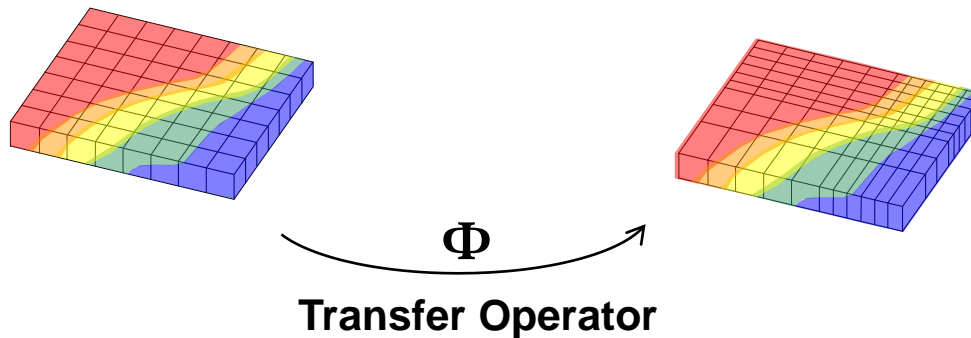
Aluminium Panel

(Benchmark 2 - Numisheet 2016)

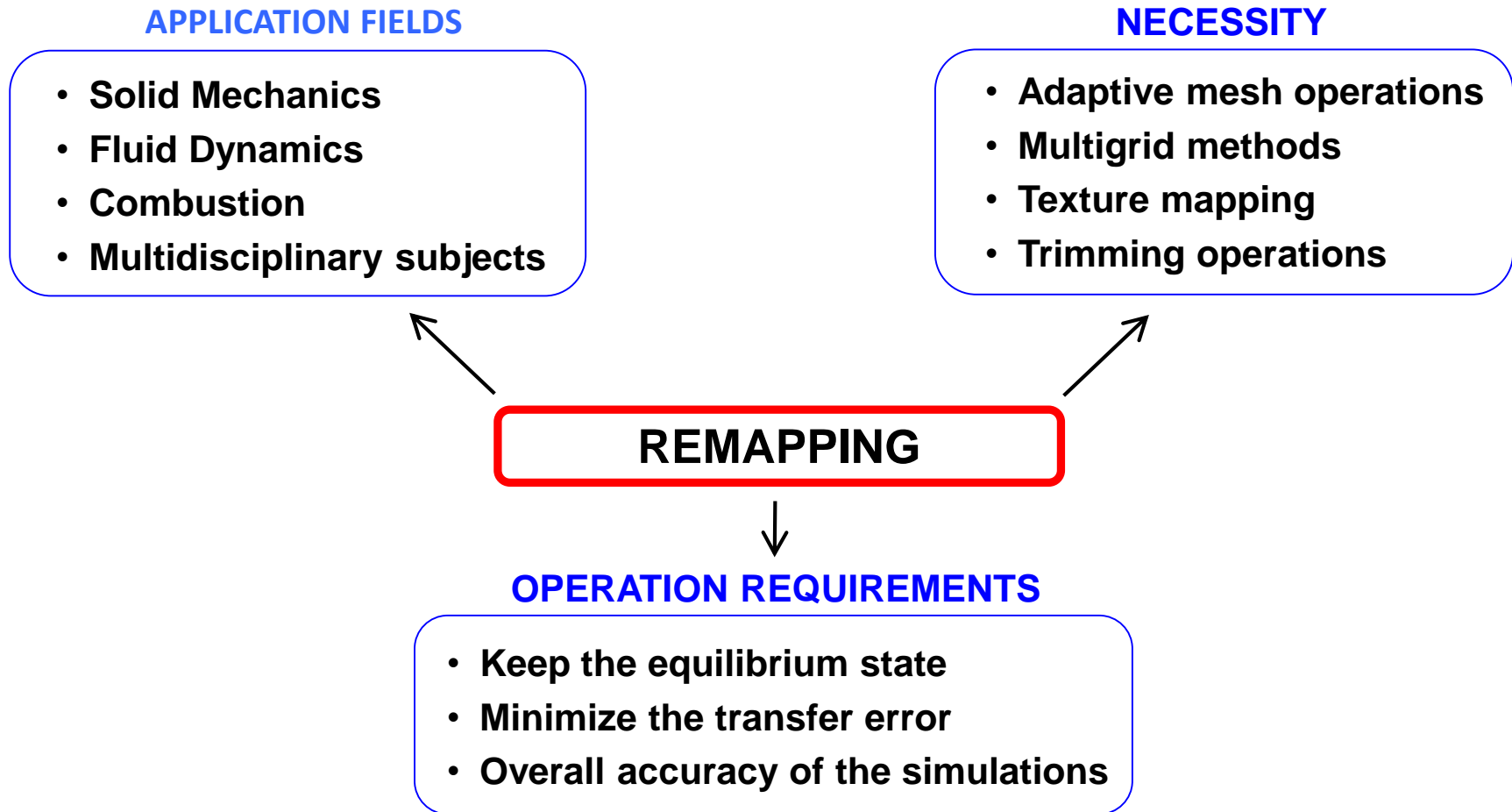
# Remapping



- **Remapping in the Nodes**  
(Nodal Variables: Force, displacement, etc.)
- **Remapping in the Gauss Points**  
(State Variables: Stress, density, etc.)

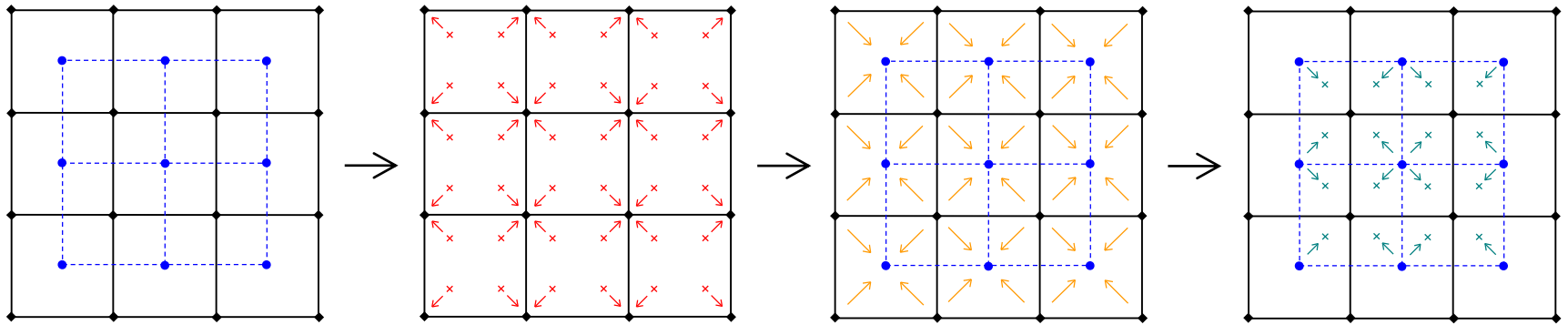


# Remapping





# Remapping methodologies



Original meshes

Extrapolation

Interpolation I

Interpolation II

$$\alpha_j = \sum_{i=1}^n \tilde{N}_{ij}(\xi, \eta, \zeta) \alpha_i$$

$$\alpha_{ig} = \sum_{j=1}^n \tilde{N}_{jig}(\xi, \eta, \zeta) \alpha_j$$

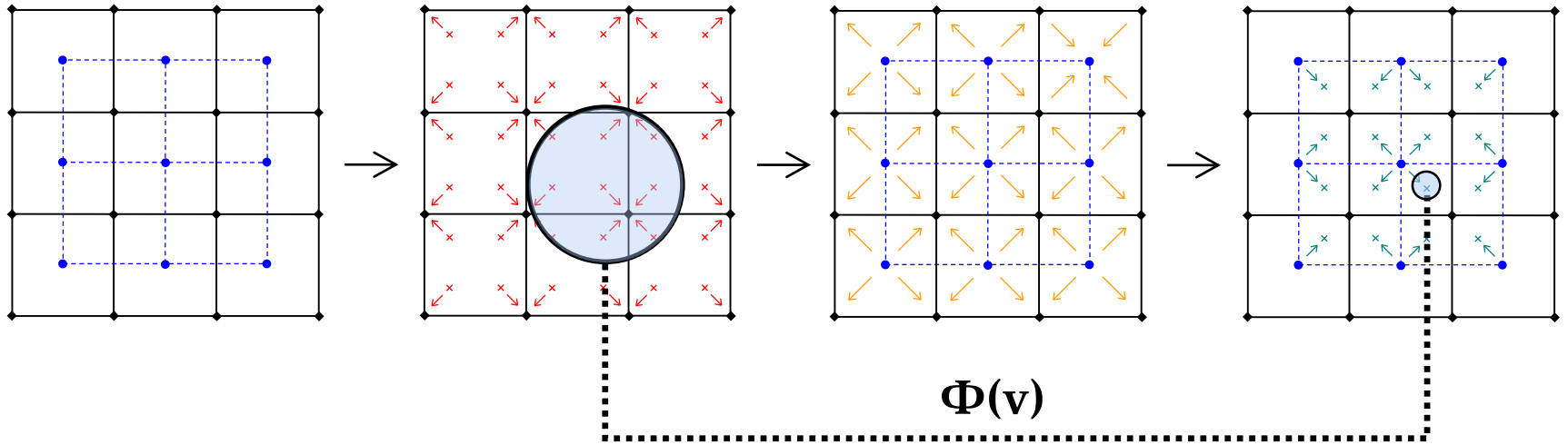
- Finite element shape functions

$$\alpha_i = \sum_{ig=1}^{ng} [\tilde{N}_{igi}(\xi, \eta, \zeta)]^{-1} \alpha_{ig}$$

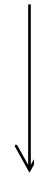
- Minimum least square

$$I(\mathbf{x}_i) = \sum_{ig}^N w(\mathbf{x}_{ig}) [\alpha(\mathbf{x}_i) - \alpha_{ig}]^2$$

# Remapping – Incremental Volumetric Remapping



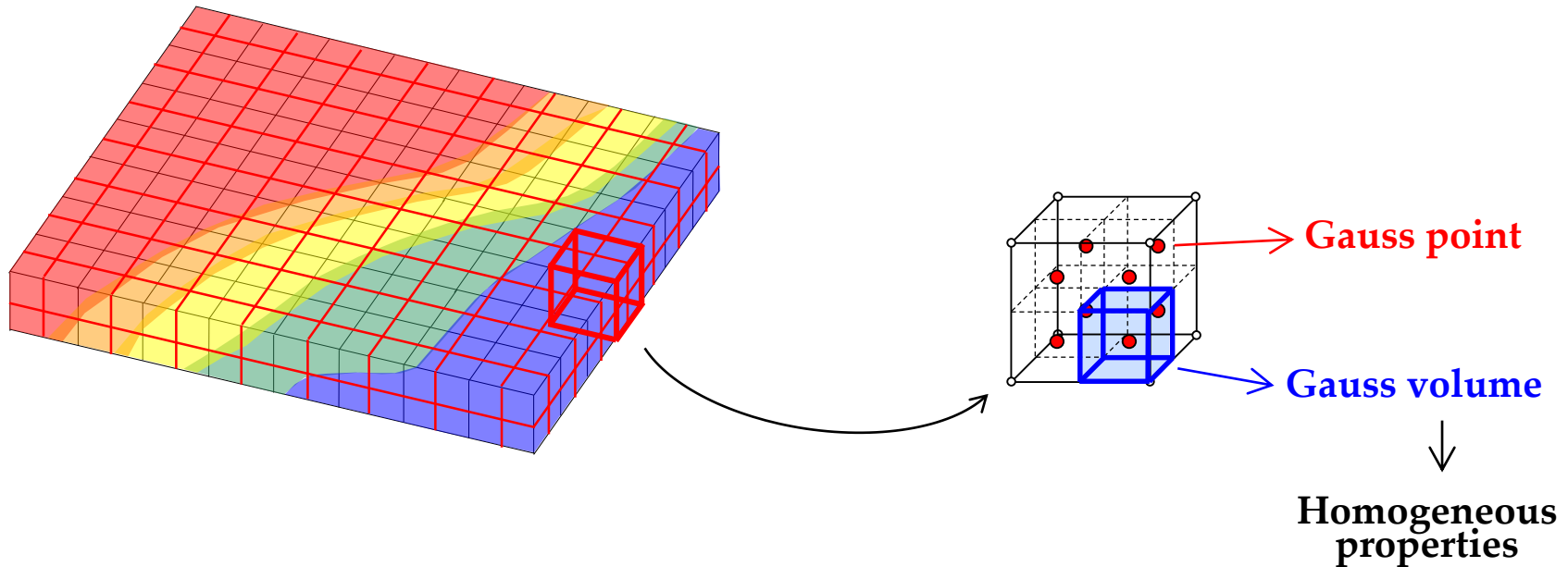
Direct transfer weighted with volume averaging



**INCREMENTAL VOLUMETRIC REMAPPING – IVR**

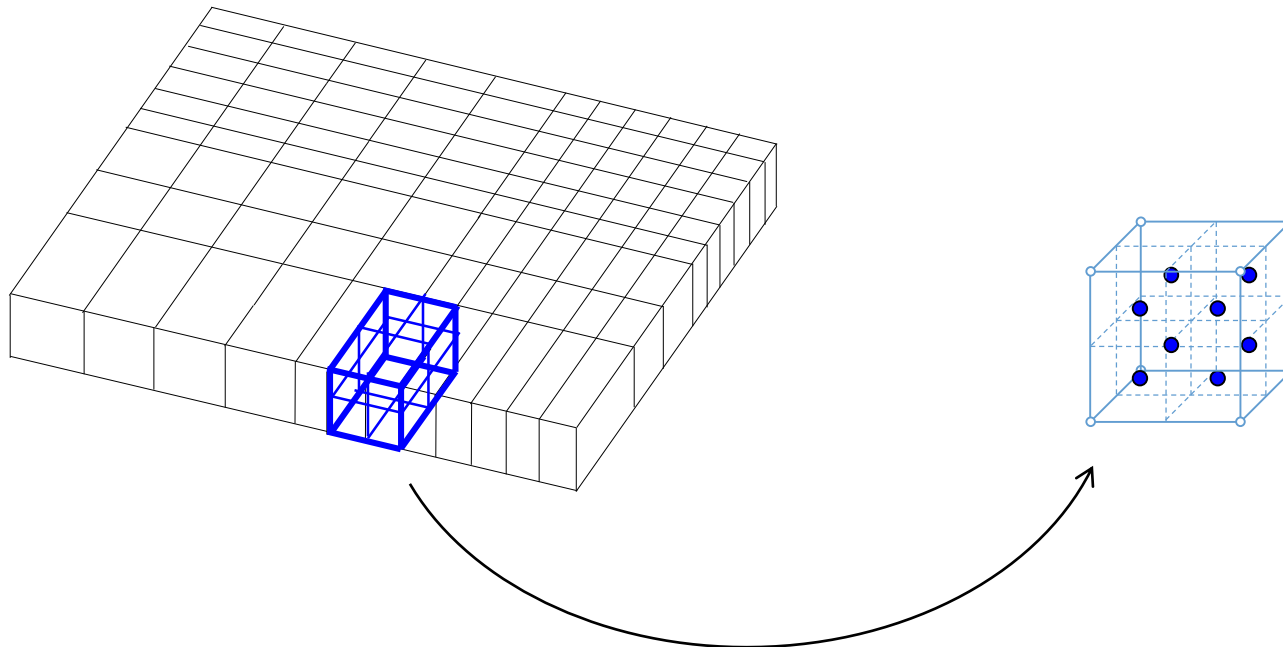
# Remapping – Incremental Volumetric Remapping

i) Step 1 – Divide all donor elements in Gauss Volumes



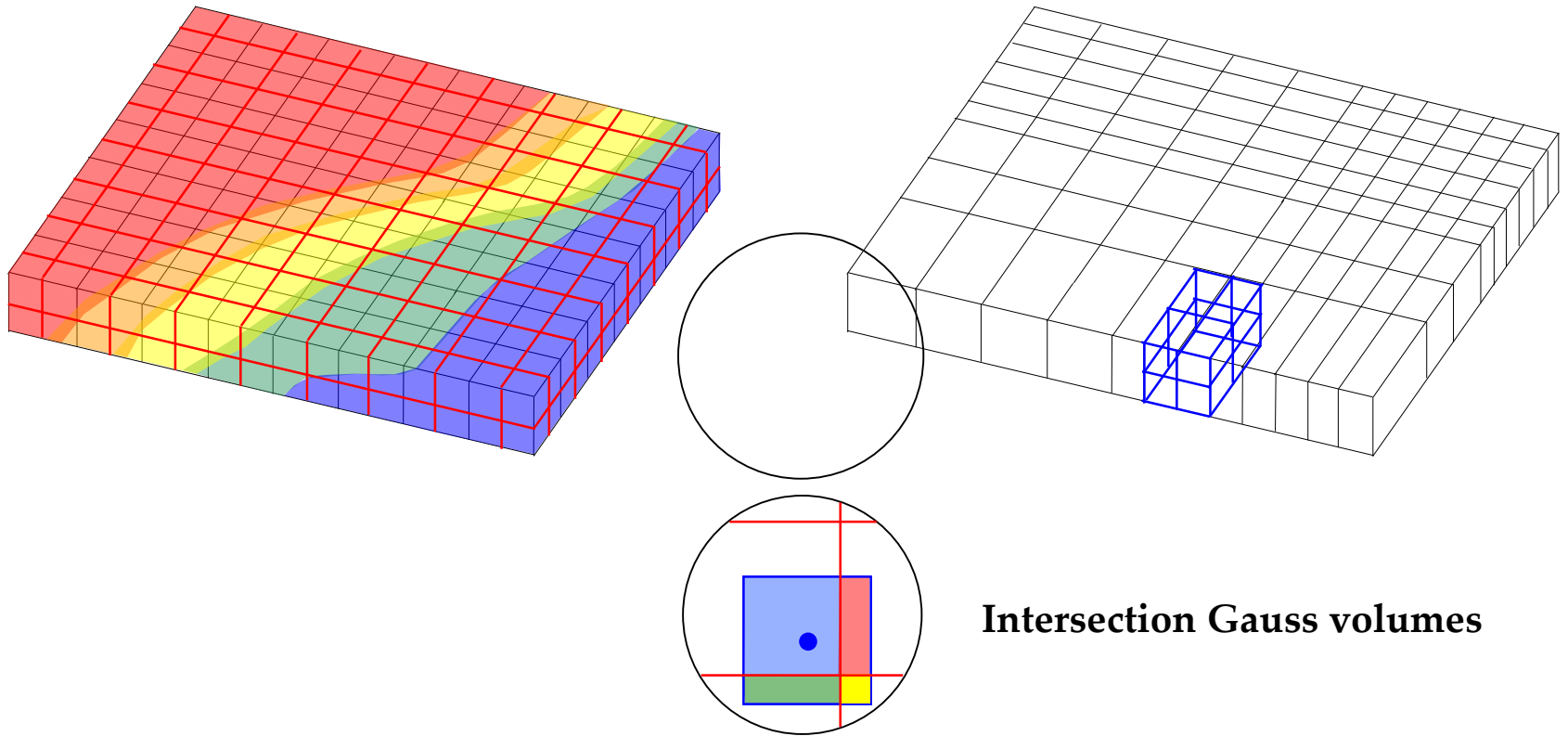
# Remapping – Incremental Volumetric Remapping

ii) Step 2 – Divide all target elements in Gauss Volumes



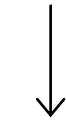
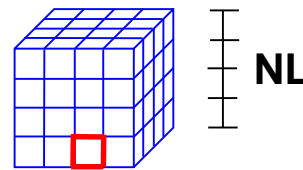
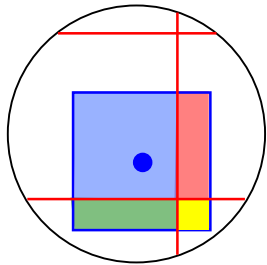
# Remapping – Incremental Volumetric Remapping

iii) Step 3 – Intersect each target Gauss volume with the donor Gauss volumes

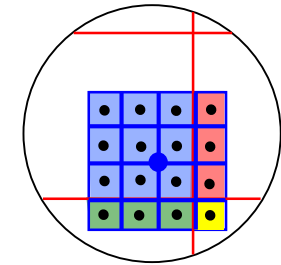


# Remapping – Incremental Volumetric Remapping

iv) Step 4 – Find the donor Gauss volume that encloses each target Gauss volume part  
 Volume division of each target Gauss volume



Gauss volume part

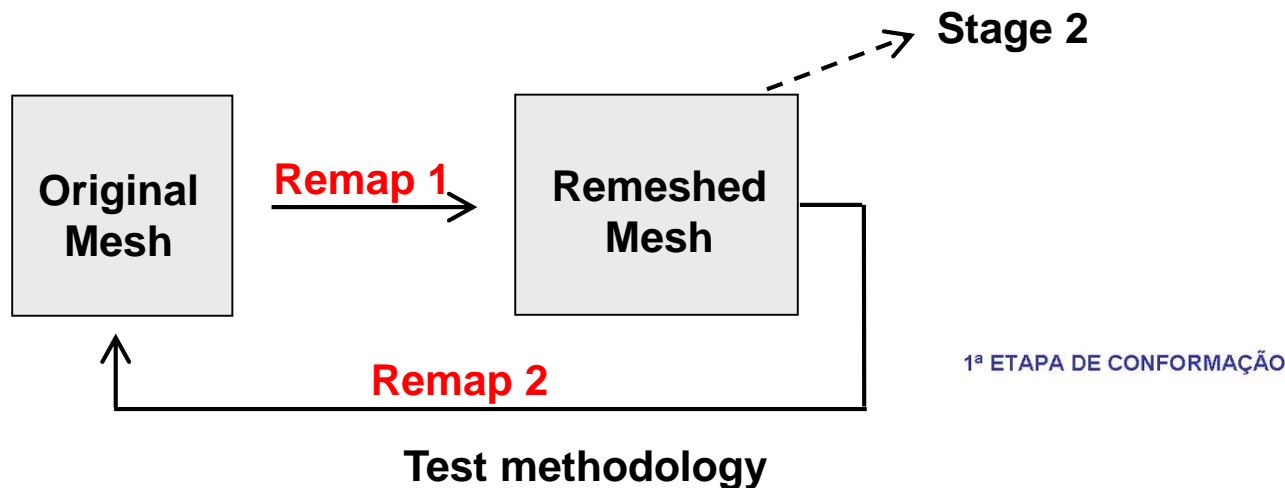


Weighted volume  
average

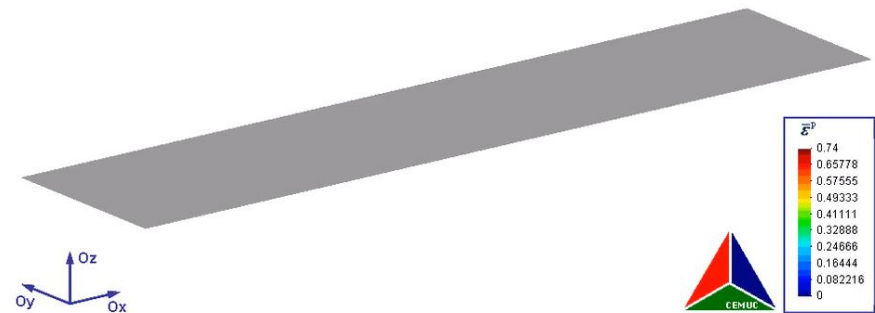
$$\alpha = \sum_{i=1}^{NG} \underbrace{\frac{\sum_{j=1}^{NL^3} iV_j}{iV_{tot}}}_{\Phi(\mathbf{v})} \alpha_i$$

$\Phi(\mathbf{v})$

# Remapping operation: Comparison of methods

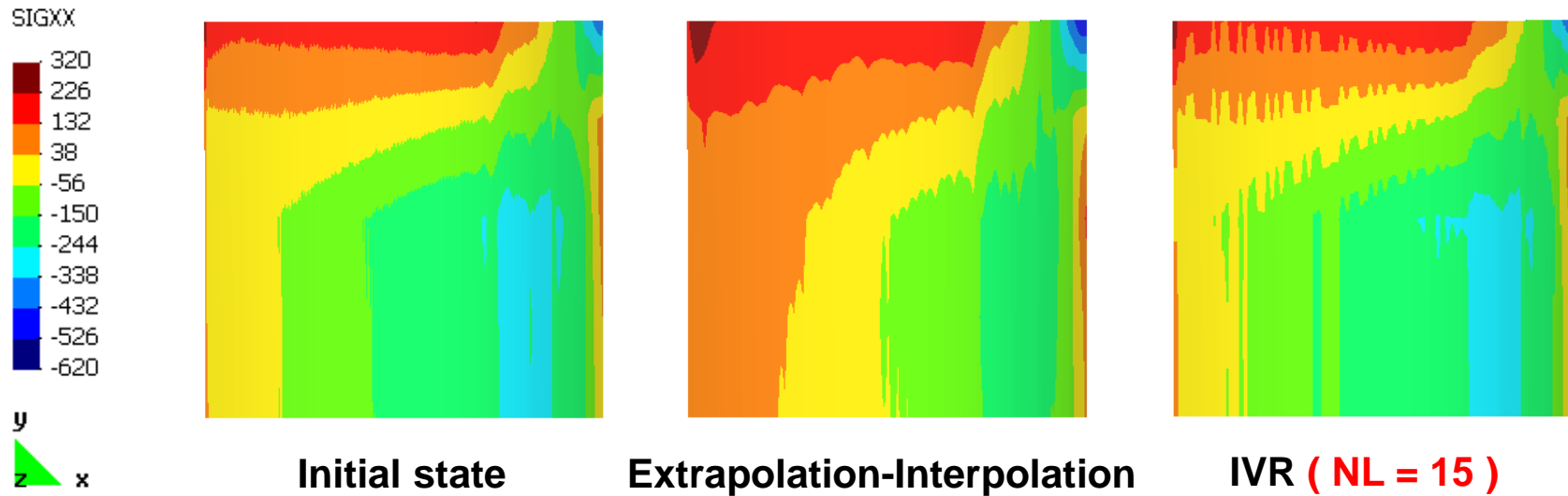


- Standard extrapolation-interpolation
- Incremental Volumetric Remapping
- State variable analysed:  $\sigma_{xx}$  stress



Channel draw/Cylindrical Cup 2-stage test  
(Benchmark 3 - Numisheet 2005)

# Remapping operation





## Remapping – Dual Kriging

- For the target Gauss point (GP),  $\mathbf{x}_t = [x \ y \ z]^T$ , the interpolation function can be decomposed in two terms

$$\alpha(\mathbf{x}_t) = d(\mathbf{x}_t) + f(\mathbf{x}_t)$$

**Drift**

$$d(\mathbf{x}_t) = \mathbf{d}^T \cdot [1 \ \mathbf{x}_t^T] = d_1 + d_2x + d_3y + d_4z$$

**Fluctuation**

$$f(\mathbf{x}_t) = \sum_{i=1}^n \lambda_i K(h_{ti}) = \sum_{i=1}^n \lambda_i K(|\mathbf{x}_t - \mathbf{x}_i|)$$

- To obtain the value of this function, it is necessary to solve a system of linear equations:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \times \mathbf{u} = \mathbf{f} \Leftrightarrow \left[ \begin{array}{ccc|cc} K(|\mathbf{x}_i - \mathbf{x}_j|) & 1 & \mathbf{x}_1^T & & \\ & \vdots & \vdots & & \\ & 1 & \mathbf{x}_n^T & & \\ \hline 1 & \cdots & 1 & 0 & \cdots & 0 \\ & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}_1 & \cdots & \mathbf{x}_n & 0 & \cdots & 0 \end{array} \right] \times \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ d_1 \\ \vdots \\ d_s \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

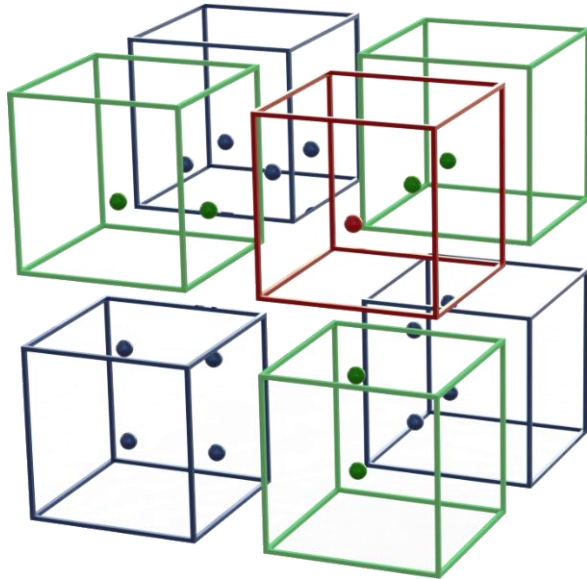
## Remapping – Dual Kriging

- For each target GP:
  - Select a set of neighbour donor GPs;
  - Evaluate the kriging coefficients and weighting factors;
  - Evaluate the state variable value in the target GP.
- Several selection algorithms for choosing the set of donor GPs were tested., based on the definition:

Name	Definition
<b>Master Node</b>	Closest node from the donor mesh to the target GP
<b>Master Element</b>	Donor mesh's element partially defined by the Master Node, containing the target GP
<b>Master GPs</b>	Donor mesh's GPs belonging to the Master Element
<b>Slave Elements</b>	Donor mesh's elements that share the Master Node, but do not contain the target GP
<b>Slave GPs</b>	Donor mesh's GPs that belong to any Slave Element, which will be considered in the Dual Kriging process.

# Remapping – Dual Kriging

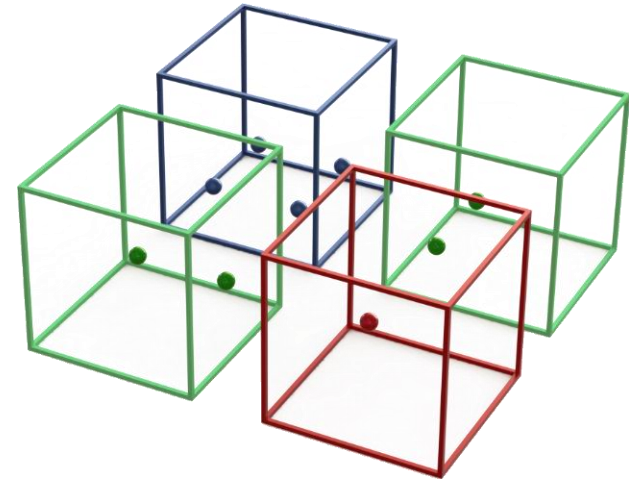
## Master-Slave method



Set of donor GPs ( $n=27$ ):

- (i) 8 of the omitted Master Element;
- (ii) 4 of each Slave Element with only one shared face (in blue);
- (iii) 2 of each Slave Element with only one shared edge (in green);
- (iv) 1 of each Slave Element with only one shared vertex (in red).

## Planar selection

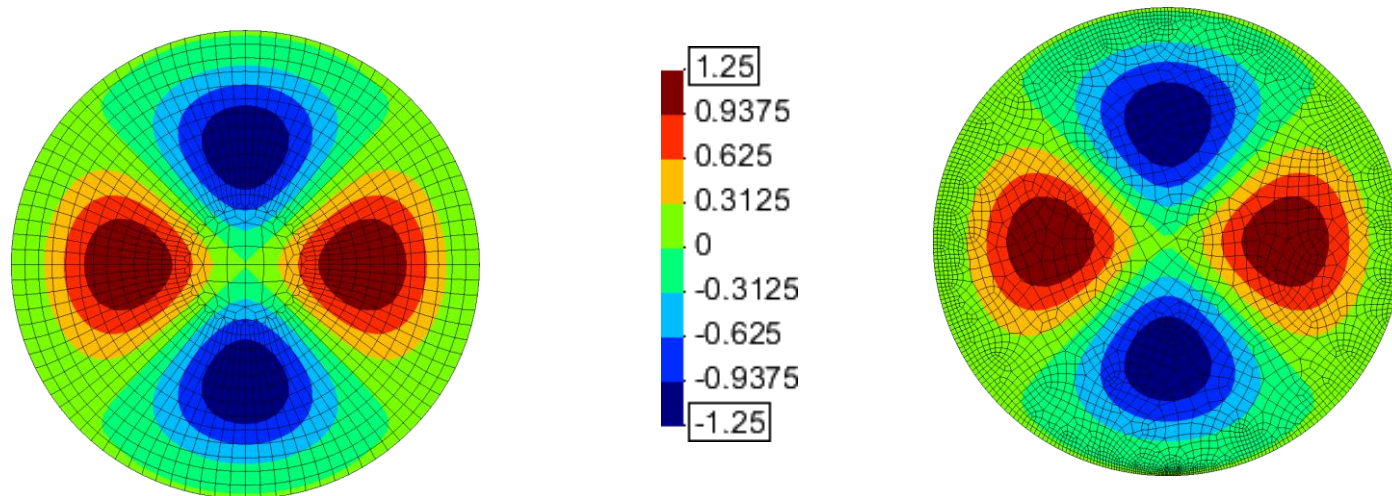


Set of donor GPs ( $n=9$ ):

- (i) 4 of the Master Element located in the same reference face (in blue);
- (ii) 2 of each Element with one shared edge with Slave the reference face (in green);
- (iii) 1 of each Slave Element with one shared vertex with the reference face (in red).

## Remapping – Dual Kriging accuracy

- The remapping is carried out in two stages, first, from the structured to the unstructured mesh (called *stage 1*) and afterwards, back to the structured mesh (called *stage 2*).
- Analytical function:  $T(r, \theta) = 20r^2(r-1)^2 \cos(2\theta)$ ,  $r = \sqrt{\frac{x^2 + y^2}{a^2}}$

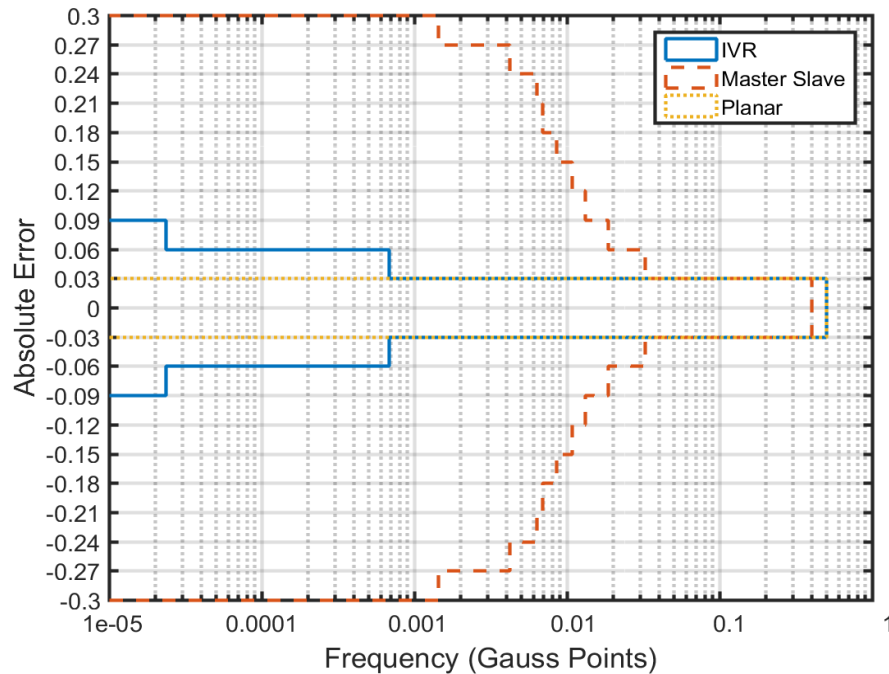


- Two FE layers through-thickness, from the top to the bottom layer, equal to:  $T$ ,  $0.1 T$ ,  $-0.1 T$ , and  $-T$ .

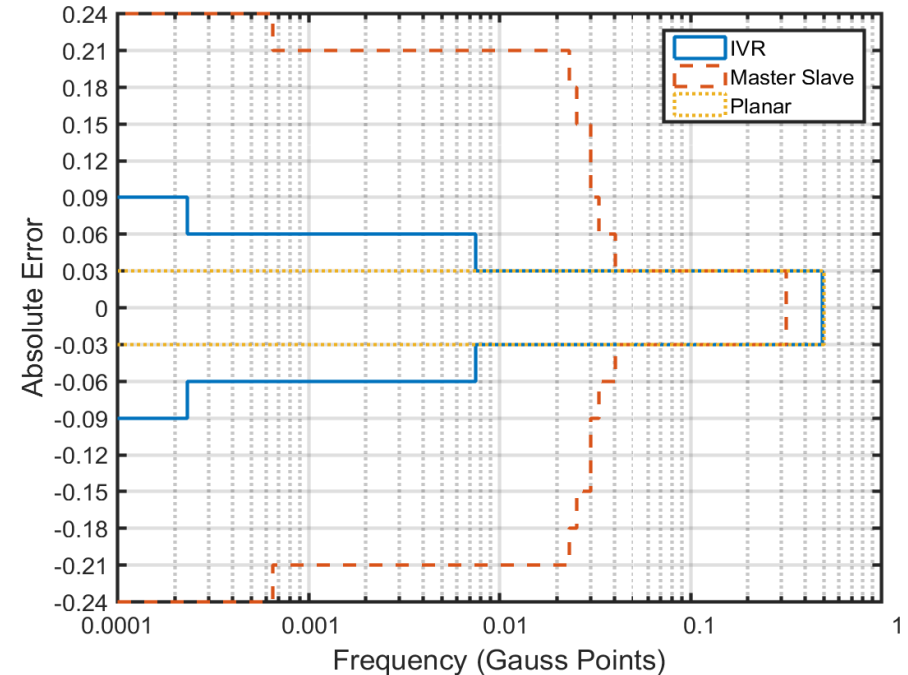
# Remapping – Dual Kriging accuracy

- Absolute error associated with the remapping method:  $E(r, \theta) = T(r, \theta) - \alpha(x_t)$

Stage 1

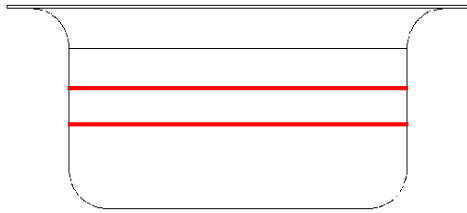


Stage 2

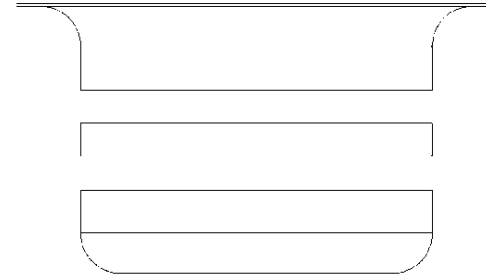


- The distribution obtained with the Master-Slave method has the highest spread and error value, particularly for *Stage 2*.

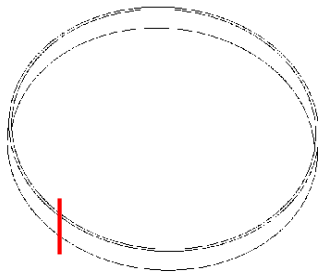
# Springback – DEMERI TEST



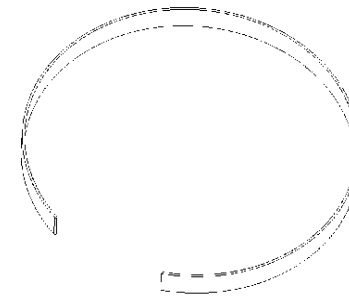
Stage I – Forming of the cup



Stage II – Cut the ring



Stage III – Split the ring

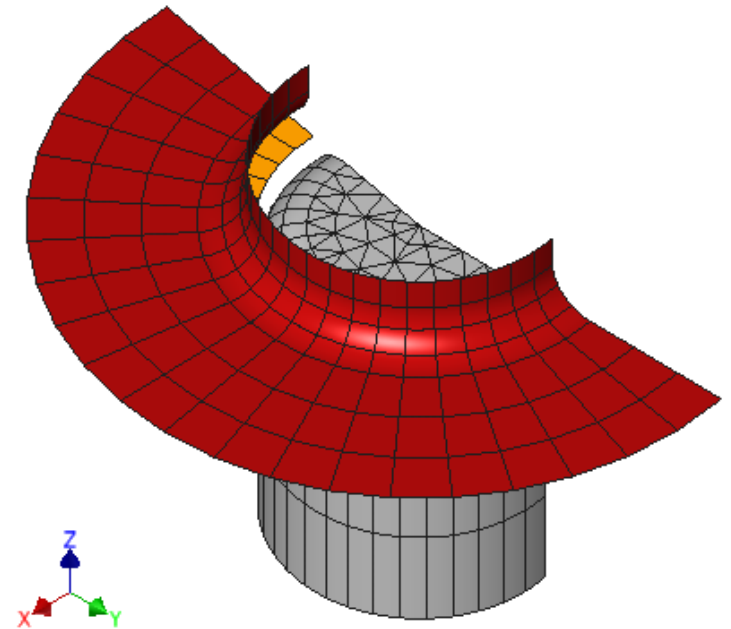
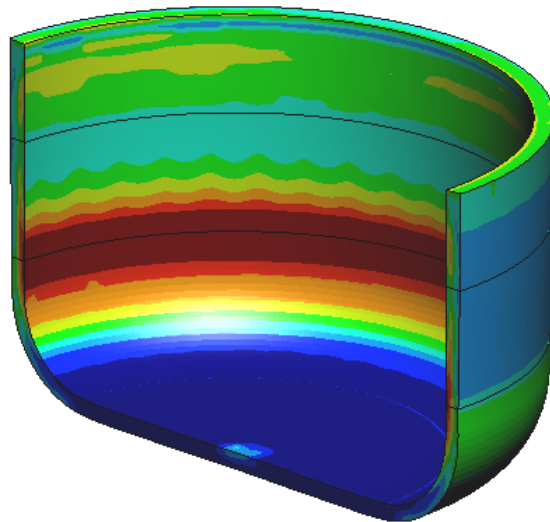
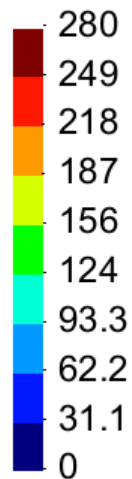


Stage IV – Open the ring

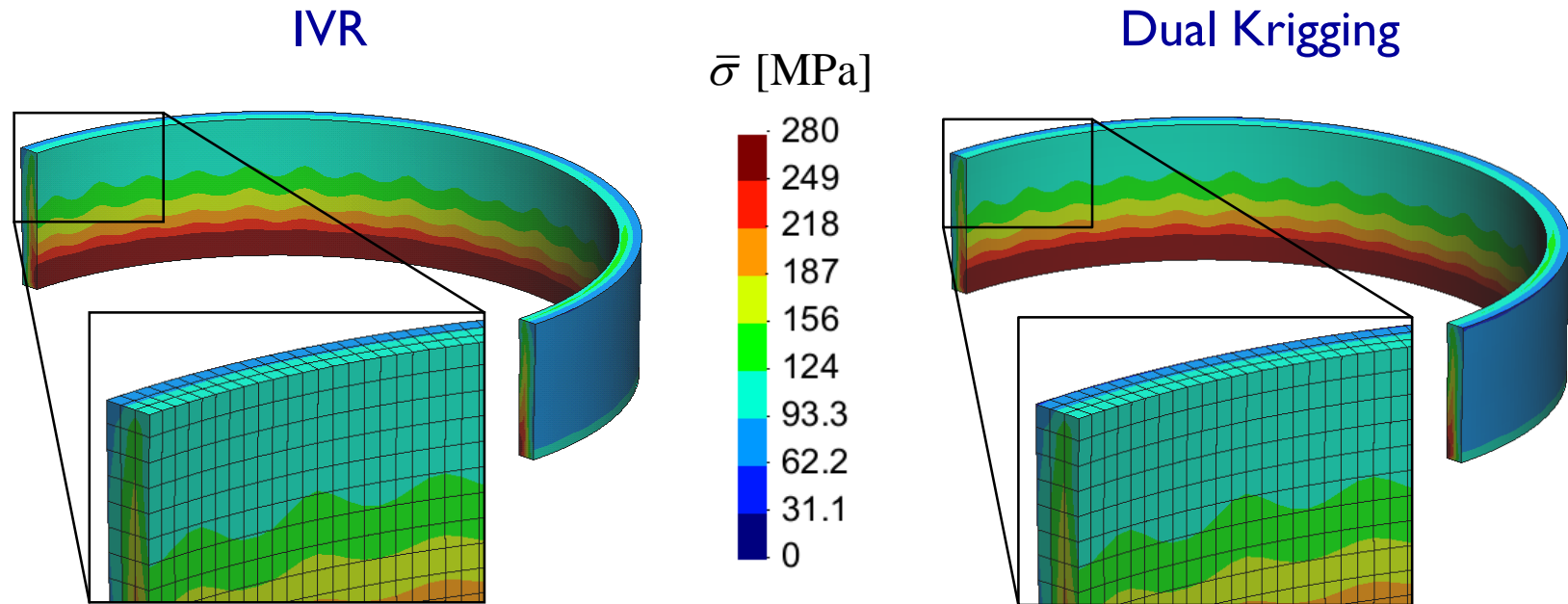
# Springback – DEMERI TEST

- AA5754-O aluminum alloy
- Blank with  $\varnothing 60$  mm and 1 mm thickness
- 1/2 of the model (symmetry conditions)
- Isotropic work hardening (voce law) and isotropic yield criterion (von Mises)

$\bar{\sigma}$  [MPa]



# Springback – DEMERI TEST



Ring opening	
4.3 mm	4.3 mm
Computational cost for the remapping [sec.]	
300	<2



## Conclusions

- This work presents the application of the Dual Kriging interpolation method as remapping scheme for FEM state variables between different finite element meshes.
- When considering a state variable distribution with gradient in all directions, combined with a small thickness dimension, a decrease of accuracy is observed for the Master-Slave method, due to an overweighting of the through-thickness component, while the Planar method kept its level of accuracy.
- The computational cost incurred for the Dual Kriging significantly lower than the IVR method.
- All remapping methods can provide results with a good level of accuracy.

## Acknowledgements

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