Tension-compression asymmetry modelling: strategies for anisotropy parameters identification.

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Agenda

- Motivation
- Objectives
- Constitutive model
  - Yield criteria
- Material parameters identification
- Cup drawing of a circular blank
  - Problem description
  - Results and discussion
- Other numerical examples
In recent years, modern industries are increasingly relying in metals with outstanding thermal and mechanical properties. Titanium, Zirconium, and Magnesium-Lithium alloys are some examples. However, these materials present challenges such as poor formability and high manufacturing costs.
Motivation

Hexagonal close-packed materials (HCP)

- Activation of single crystal deformation mechanisms
- Slip with pronounced non-Schmidt effect

Tension-compression asymmetry
Motivation

Numerical Simulation!

Modelling → Functions capable of modelling T-C

Characterization → Buckling effects (compression stress states)…
Cazacu Barlat Plunkett, 2006, yield criterion allows modelling of tension-compression asymmetry
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Objectives

- Anisotropy parameters identification for two yield criteria
  - CPB06
  - YLD91

- Influence of accounting for tension-compression asymmetry in the numerical simulation of a cup drawing
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Yield criteria

CPB06

- Equivalent stress given by

\[
\bar{\sigma} = B \left[ \left( |s_1| - k \right)^a + \left( |s_2| - k \right)^a + \left( |s_3| - k \right)^a \right]^{1/a}
\]

\(s_1, s_2\) and \(s_3\) are the principal stresses of \(S = C\sigma'\) and

\[
B = \frac{1}{\left[ \left( |\phi_1| - k \right)^a + \left( |\phi_2| - k \right)^a + \left( |\phi_3| - k \right)^a \right]^{1/a}}
\]

\(k\) and \(a\) are material parameters
Yield criteria

CPB06

$$B = \left[ \frac{1}{\left( |\phi_1| - k_1 \right)^a + \left( |\phi_2| - k_2 \right)^a + \left( |\phi_3| - k_3 \right)^a} \right]^{1/a}$$

where

$$\begin{align*}
\phi_1 &= \left( \frac{2}{3} \right) C_{11} - \left( \frac{1}{3} \right) C_{12} - \left( \frac{1}{3} \right) C_{13} \\
\phi_2 &= \left( \frac{2}{3} \right) C_{21} - \left( \frac{1}{3} \right) C_{22} - \left( \frac{1}{3} \right) C_{23} \\
\phi_3 &= \left( \frac{2}{3} \right) C_{31} - \left( \frac{1}{3} \right) C_{32} - \left( \frac{1}{3} \right) C_{33}
\end{align*}$$

$$\mathbf{C} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}$$
Yield criteria

YLD91

- Equivalent stress given by

\[
\bar{\sigma} = \left\{ \frac{1}{2} \left[ |s_1 - s_2|^m + |s_2 - s_3|^m + |s_1 - s_3|^m \right] \right\}^{\frac{1}{m}}
\]

\(s_1, s_2\) and \(s_3\) are the principal stresses of \(S = L\sigma'\) and

\[
L = \begin{bmatrix}
\frac{c_2 + c_3}{3} & -\frac{c_3}{3} & -\frac{c_2}{3} & 0 & 0 & 0 \\
-\frac{c_3}{3} & \frac{c_3 + c_1}{3} & -\frac{c_1}{3} & 0 & 0 & 0 \\
-\frac{c_2}{3} & -\frac{c_1}{3} & \frac{c_1 + c_2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & c_4 & 0 & 0 \\
0 & 0 & 0 & 0 & c_5 & 0 \\
0 & 0 & 0 & 0 & 0 & c_6 \\
\end{bmatrix}
\]

Note:
\(m = 6\) BCC
\(m = 8\) FCC
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The yield criterion should reproduce the material’s mechanical behavior

\[
F(\mathbf{A}) = \sum_{\theta=0}^{90} w_{\sigma_{\theta}^T} \left( \sigma_{\theta}^{Y_T}(\mathbf{A})/\sigma_{\theta}^{Y_T} - 1 \right)^2 + \sum_{\theta=0}^{90} w_{\sigma_{\theta}^C} \left( \sigma_{\theta}^{Y_C}(\mathbf{A})/\sigma_{\theta}^{Y_C} - 1 \right)^2 + \sum_{\theta=0}^{90} w_{r_{\theta}} \left( r_{\theta}(\mathbf{A})/r_{\theta} - 1 \right)^2 \\
+ w_{\sigma_b} \left( \sigma_b(\mathbf{A})/\sigma_b - 1 \right)^2 + w_{r_b} \left( r_b(\mathbf{A})/r_b - 1 \right)^2
\]

\(\mathbf{A}\) - set of anisotropy parameters
\(\sigma_{\theta}^{Y_T}, \sigma_{\theta}^{Y_C}\) - experimental yield stresses in tension and compression
\(r_{\theta}\) - experimental \(r\)-values
\(\sigma_b\) - experimental biaxial yield stress
\(r_b\) - experimental disc compression test \(r\)-value
Anisotropy parameters identification

2090-T3 aluminum

Figure 1. Experimental and predicted (a) r-value and (b) normalized yield stress.

- CPB06 shows a different behavior in tension and compression
- Though not very flexible.
2090-T3 aluminum

![Predicted yield surfaces](image)

**Figure 2.** Predicted yield surfaces.

- Neither yield criterion accurately describes the biaxial point.
- Ratios are only an indication.

<table>
<thead>
<tr>
<th>Table 1. Ratios obtained for the three principal axis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\sigma_1^T/\sigma_1^C)$</td>
</tr>
<tr>
<td>Experimental</td>
</tr>
<tr>
<td>CPB06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Experimental and numerically predicted biaxial tensile values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_b$</td>
</tr>
<tr>
<td>Experimental</td>
</tr>
<tr>
<td>YLD91</td>
</tr>
<tr>
<td>CPB06</td>
</tr>
</tbody>
</table>
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Problem description

Figure 3. Schematic of the cup drawing and main dimensions.

Figure 4. In-plane blank sheet discretization.
## Results and discussion

- The rim response in the Rolling Direction will be dictated by the material properties in the Transverse direction.

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**Figure 5.** Deformation of an element on the flange: (a) stress states on the flange and (b) stress states on the yield surface (adapted from Yoon et al. 2011).
Results and discussion

Figure 6. Comparison between experimental and numerically predicted cup height vs. angle from rolling direction.

- CPB06 has a lower earing profile, coherent with latter yielding – higher yield stress in TD.
  - Also higher $r$-value at TD.

Figure 7. Numerically predicted punch force and blank holder displacement with punch displacement.
**Results and discussion**

![Graphs showing evolution of strain ratio with punch displacement](image)

**Figure 8.** Evolution, with the punch displacement, of $\varepsilon_r/\varepsilon_t$ (solid lines) and $r$-values (dashed lines) estimated with (a) YLD91 and (b) CPB06 yield criteria.

- Both yield criteria predicted and calculated values are in very good agreement.
- Low blank-holder force allows not altering the stress state in the flange.
Results and discussion

Figure 9. Evolution, with the punch displacement, of $\sigma_\theta/Y$ (solid lines) and ratio between yield stress (dashed lines) estimated with (a) YLD91 and (b) CPB06 yield criteria.

- Both yield criteria predicted and calculated values are in very good agreement.
- Low blank-holder force allows not altering the stress state in the flange.
Results and discussion

- CPB06 predicts the difference between RD and TD.

**Figure 10.** Evolution of the predicted and measured strain, for the YLD91 and CPB06 yield criteria, regarding the rolling and transverse directions.
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Other numerical examples

- Four-point bending test (Zirconium)
Mg-AZ31 magnesium alloy

**Example of an identification using only tensile results**

\[ \sigma_{xx}/\sigma_T \]

\[ r \]

<table>
<thead>
<tr>
<th>Exp</th>
<th>Id</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_b )</td>
<td>213,31</td>
</tr>
<tr>
<td>( r_b )</td>
<td>0,579</td>
</tr>
</tbody>
</table>
### Other numerical examples

- **Four-point bending test (Zirconium)**

**Figure 9.** Evolution of the predicted and measured strain, for the YLD91 and CPB06 yield criteria, regarding the rolling and transverse directions.

\[
\text{Isotropic material } (\mathbf{C} = \mathbf{I}) \text{ and } k = 0
\]
Other numerical examples

- Four-point bending test (Zirconium)

\[ \frac{\sigma_{\text{RD}}^T}{\sigma_{\text{RD}}^C} = 0.88; \quad \frac{\sigma_{\text{TD}}^T}{\sigma_{\text{TD}}^C} = 0.96; \quad \frac{\sigma_{\text{ND}}^T}{\sigma_{\text{ND}}^C} = 1.40 \]
Other numerical examples

Four-point bend test (Zirconium) [2]

Green Lagrange strain [-]

\[ E_{xx} \] \text{RD} \quad \text{TD} \quad \text{ND}
\[ E_{yy} \] \text{RD} \quad \text{TD} \quad \text{ND}
\[ E_{zz} \] \text{RD} \quad \text{TD} \quad \text{ND}
The authors gratefully acknowledge the financial support of the Portuguese Foundation for Science and Technology (FCT) under projects with reference PTDC/EMS-TEC/0702/2014 (POCI-01-0145-FEDER-016779) and PTDC/EMS-TEC/6400/2014 (POCI-01-0145-FEDER-016876) by UE/FEDER through the program COMPETE 2020. The first author is also grateful to the FCT for the PhD grant SFRH/BD/98545/2013.